Dynamical collapse of trajectories

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Dry friction causes distinct trajectories to collapse onto a single point in finite time, such that not all trajectories can be continued in the backward direction of time. This phenomenon is captured in simple friction models such as Amontons-Coulomb's friction law. A result is that transversal homoclinic orbits generate chaotic saddles with forward dynamics that is qualitatively different from the backward dynamics. Namely, the forward dynamics is restricted to an unstable set, of which parts may collapse onto each other. In this manner, friction destroys the complexity of the forward dynamics by generating a unique horseshoe-like topology.

Focussing on the neighbourhood of a transversal homoclinic orbit emanating from a set of non-isolated equilibrium points, this dynamics is described with a map similar to the horseshoe map of Smale. This map is shown to be topologically conjugate to a symbolic dynamics. The collapse of trajectories is introduced in the symbolic dynamics by defining this dynamics as a shift on a quotient space of the standard symbolic state space with infinite strings of two symbols. Using the mentioned conjugacy, it is proven that a limit set exists that contains an infinite number of periodic orbits.