



# FIFTH SYMPOSIUM OF THE EUROPEAN NETWORK FOR NONSMOOTH DYNAMICS (ENNSD)



LIEGE, SEPTEMBER 8-9, 2016



## SCIENTIFIC PROGRAM DAY 1

THURSDAY, Sept. 8th 2016	
9:00	Registration
9:30	Opening
9:40	<b>B. Brogliato, N.S. Nguyen: "Comparison of rigid body multiple contact/impact models"</b>
10:40	Coffee break
11:00	<b>J. Kleinert, B. Simeon: "Highly accurate nonsmooth contact dynamics for the large-scale simulation of granular material"</b>
12:00	Lunch
13:20	<b>R.I. Leine: "A mixed shooting harmonic balance method for nonsmooth mechanical systems"</b>
14:20	Small break
14:30	<b>F. Dubois: "Mixed implicit and explicit formulation of contact dynamics"</b>
15:30	Coffee break
15:50	<b>G. de Saxcé: "A survey of bipotential-based variational and numerical methods"</b>
18:00	Visit of the Curtius brewery in Liège
19:30	Dinner in Liège

### DESCRIPTION

#### Comparison of rigid body multiple contact/impact models (9:40)

[Bernard Brogliato](#) (INRIA Grenoble, France), [Ngoc Son Nguyen](#) (University of Nantes, France)

In this preliminary work we present the comparison of three multiple impact laws: Moreau's law, the binary collision model, and the LZB model. Taking a chain of three aligned balls, and considering the LZB model as a "reference", the domains of validity of the binary and of Moreau's laws are determined numerically when mass ratios, stiffness ratios, elasticity coefficients are varied. The two criteria are the energy dissipation, and most importantly the energy dispersion (that is, how the initial kinetic energy is dispersed through the chain, as a result of wave effects). The objective is to extend this analysis to longer chains.

#### Highly accurate nonsmooth contact dynamics for the large-scale simulation of granular material (11:00)

[Jan Kleinert](#) (Fraunhofer Institute for Algorithms and Scientific Computing, St. Augustin, Germany), [Bernd Simeon](#) (TU Kaiserslautern, Germany)

For the large-scale simulation of granular material, the Nonsmooth Contact Dynamics Method (NSCD) is examined. First, the equations of motion of nonsmooth mechanical systems are introduced and classified as a Differential Variational Inequality (DVI) that has a structure similar to Differential-Algebraic Equations (DAEs). Using a Galerkin projection in time, we derive nonsmooth extensions of the SHAKE and RATTLE time integration schemes. A matrix-free Interior Point Method (IPM) is used for the complementarity problems that need to be solved in each time step. We demonstrate that in this way, the NSCD approach yields highly accurate results and is competitive compared to the Discrete Element Method (DEM).

## A mixed shooting harmonic balance method for nonsmooth mechanical systems (13:20)

Remco I. Leine (Universität Stuttgart, Germany)

In this talk we present a mixed shooting – harmonic balance method for large linear mechanical systems with local nonlinearities. The standard harmonic balance method (HBM) approximates the periodic solution in frequency domain and is very popular as it is well suited for large systems with many states. Local nonlinearities cannot be evaluated directly in the frequency domain. The standard HBM performs an inverse Fourier transform, then calculates the nonlinear force in time domain and subsequently the Fourier coefficients of the nonlinear force. The disadvantage of the HBM is, that strong nonlinearities are poorly represented by a truncated Fourier series. In contrast, the shooting method operates in time-domain and relies on numerical time-simulation. Set-valued force laws such as dry friction or other strong nonlinearities can be dealt with if an appropriate numerical integrator is available. The shooting method, however, becomes infeasible if the system has many states. The proposed mixed shooting–HBM approach combines the best of both worlds.

## Mixed implicit and explicit formulation of contact dynamics (14:30)

Frédéric Dubois (Université de Montpellier, France)

The Non-Smooth Contact Dynamics approach aims at solving frictional contact problems without regularization nor penalization techniques [1]. In this framework both dynamics and contact problems were solved by implicit algorithms. The software developed called LMGC90 permits to solve coupled multi physics problems with various constitutive models and interaction laws [2].

The fracture of heterogeneous materials is modeled using a Cohesive Zone Models approach [3] implemented in a LMGC90-based software called Xper [4]. This method needs very small time steps, smaller than one verifying the Courant-Friedrich-Levy criterion for explicit scheme. Thus, to reduce computational cost while keeping results accuracy, we developed an hybrid method based on an explicit integration of dynamics and an implicit description of contact solved by a non linear Gauss-Seidel algorithm. High Performance Computing may be addressed by a Domain Decomposition Strategy based on domain overlapping multithreaded.

The numerical strategy is applied to the so called Nooru Mohamed test case. Results are similar for implicit and explicit approach, and also for sequential and multithreaded computations. Implementation is validated by analyzing balance energy. The insights of numerical algorithms will be exposed during the presentation.

### References

- [1] M. Jean. The non-smooth contact dynamics method. *Comput. Methods Appl. Mech. Engrg.*, 177:235–257, 1999.
- [2] LMGC90. <http://www.lmgc.univ-montp2.fr/LMGC90/LMGC90/>, 2013.
- [3] M. Jean, V. Acary, and Y. Monerie. Non-smooth contact dynamics approach of cohesive materials. *Phil. Trans. R. Soc. London A*, 359:2497–2518, 2001.
- [4] F. Perales, F. Dubois, Y. Monerie, B. Piar, and L. Stainier. A nonsmooth contact dynamics based multi-domain solver. *Eur. J. Comp. Mech.*, 19:389–417, 2010.

## A survey of bipotential-based variational and numerical methods (15:50)

Géry de Saxcé (Université de Lille, France)

In Non Smooth Mechanics, an important tool is the subdifferential for constructing multivalued constitutive laws in the form of a differential inclusion:

$$\mathbf{v} \in \partial\varphi(\mathbf{f}) ,$$

where  $\mathbf{v}$  and  $\mathbf{f}$  are dual generalized velocity and force variables and  $\varphi$  is a convex and lower semicontinuous potential. They are called normality rules or associated laws. The variational calculus is extended but with some infringements. The functionals being convex but not necessarily differentiable, their minima are characterized by inequations. Moreover, the potential  $\varphi$  and its polar function  $\varphi^*$  satisfy a fundamental relation, Fenchel's inequality:

$$\forall \mathbf{v}', \mathbf{f}', \quad \varphi(\mathbf{v}') + \varphi^*(\mathbf{f}') \geq \langle \mathbf{v}', \mathbf{f}' \rangle .$$

But a new obstacle arises: some of the constitutive laws are non-associated! A response proposed first in [4] consists in constructing a function of two variables, bi-convex and lower semicontinuous satisfying an inequality generalizing Fenchel's one:

$$\forall \mathbf{v}', \mathbf{f}', \quad b(\mathbf{v}', \mathbf{f}') \geq \langle \mathbf{v}', \mathbf{f}' \rangle .$$

We call it a bipotential. Physically, it represents the dissipation. The constitutive law turns out to be associated but in a weak form of an implicit relation between dual variables:

$$\mathbf{v} \in \partial b(\mathbf{v}, \bullet)(\mathbf{f}) .$$

The applications to solid Mechanics are various: Coulomb's friction law, non-associated Drucker-Prager and Cam-Clay models in Soil Mechanics, cyclic Plasticity and Viscoplasticity of metals with non linear kinematical hardening rule, Lemaitre's damage law (for more details, see reference [2]).

And the variational calculus? It can be extended thanks to a new concept, the bifunctional. The dual variational principles remain but they are coupled. The bound theorems of the limit analysis and the plastic shakedown theory [7] can be reformulated in a broader framework, precisely by means of the normality rule, weak but recovered [2]. In the numerical viewpoint, the bipotential method suggests new algorithms, fast but robust, as well as variational errors estimator assessing the accurateness of the finite element mesh. Applications to the contact Mechanics [6], the Dynamics of granular materials [5] and the Plasticity [1,3] illustrate the relevancy of this approach.

### References

- [1] Berga, A., Mathematical and numerical modeling of the non-associated plasticity of soils—Part 2: Finite element analysis, *International Journal of Non-Linear Mechanics*, 47, pp. 36-45 (2012).
- [2] Dang Van, K., de Saxcé, G., Maier, G., Polizzotto, C., Ponter, A., Siemaszko, A., Weichert, D., Inelastic Behaviour of Structures under Variable repeated Loads, D. Weichert, G. Maier, Eds., CISM International Centre for Mechanical Sciences, Courses and Lectures, No. 432, Springer, Wien, New York.(2002).
- [3] Cheng, L., Yun Jia, Oueslati, A., de Saxcé, G., Kondo, D. Bipotential-based limit analysis and homogenization of non-associated porous plastic materials, *Journal of the Mechanics and Physics of Solids*, 77, pp. 1-26 (2015).
- [4] de Saxcé, G., Feng, Z.Q., New inequation and functional for contact with friction: the implicit standard material approach, *International Journal of Mechanics of Structures and Machines*, Vol. 19, N°3, pp. 301-325 (1991).
- [5] Fortin, J., Millet, O., de Saxcé, G., Numerical simulation of granular materials by an improved discrete element method, *International Journal for Numerical Methods in Engineering*, Volume 62, pp. 639-663 (2004).
- [6] Hjjaj, M., Feng, Z.Q., de Saxcé, G., Mróz, Z., Three dimensional finite element computations for frictional contact problems with non-associated sliding rule, *International Journal for Numerical Methods in Engineering*, Volume 60, N°12, pp. 2045-2076 (2004).
- [7] Save, M.A., Massonnet, C.E., de Saxcé, G., Plastic limit analysis of plates, shells and disks. Elsevier, New York (1997).

## SCIENTIFIC PROGRAM DAY 2

FRIDAY, Sept. 9th 2016	
9:00	<b>P. Glendinning: "Attractors of piecewise smooth maps: what we know and how we know it"</b>
10:00	Small break
10:10	<b>A. Tasora: "On the efficient software implementation of solvers for complementarity problems in nonsmooth dynamics"</b>
11:10	Coffee break
11:30	<b>V. Acary: "How to solve efficiently 3D frictional contact problem?"</b>
12:30	Lunch
13:50	<b><u>N. Van de Wouw</u>, M. Baumann, J.J.B. Biemon, R.I. Leine: "Synchronization of mechanical systems with a geometric unilateral constraint"</b>
14:50	Small break
15:00	<b>O. Brüls: "Simulation of grasping tasks using the nonsmooth generalized-<math>\alpha</math> method"</b>
16:00	Closing
	End of the symposium

## DESCRIPTION

### **Attractors of piecewise smooth maps: what we know and how we know it (9:00)**

Paul Glendinning (University of Manchester, U.K.)

I will review some of the methods and results available to describe attractors of piecewise smooth maps. The structure of non-wandering sets, the existence of invariant measures, and the dimension of bifurcating attractors will be addressed with varying levels of generality. I believe that the methods available to prove these results are just as important as the results themselves, and these techniques (and hopes for extensions of these techniques) will be the main driver of the talk.

### **On the efficient software implementation of solvers for complementarity problems in nonsmooth dynamics (10:10)**

Alessandro Tasora (Università degli Studi di Parma, Italy)

The solution of complementarity problems arising in nonsmooth dynamics poses a challenge that motivates our research on fast and robust numerical methods. This presentation will start with an introduction about the cone-complementarity formulation adopted in our open-source software project, Chrono::Engine, which aims at the simulation of systems with a massive amount of frictional contacts. In this context, we will also cover collateral achievements that we pursued, such as elasto-visco-plastic contacts and cohesive effects with variational inequalities. In a second part, we will discuss a solver based on a modified version of the spectral projected gradient method. Convergence, preconditioning, software implementation, optimizations and opportunities will be shown, and benefits, limits and open problems will be described. A comparison with another solver based on the Nesterov method will be reviewed too. Finally, in the last part we will show examples of engineering problems that profited from the nonsmooth dynamics approach discussed here. In detail, we will show applications to the simulation of waste processing flows, to the simulation of masonry and rocking-block structures, and to the simulation of vehicle mobility. Open questions and future developments will follow the conclusion.

## How to solve efficiently 3D frictional contact problem? (11:30)

Vincent Acary (INRIA Chile)

In this talk, we want to discuss possible numerical solution procedures for the discrete frictional contact problem that is at the heart of the simulation of mechanical systems with 3D Coulomb's friction and unilateral constraints. We will list several algorithms that have been previously developed for solving the second order cone complementarity problem (SOCCP) mainly based variational inequality and nonsmooth equations reformulations. On one hand, we will show that algorithms based on Newton methods for nonsmooth equations solve quickly the problem when they succeed, but suffer from robustness issues mainly if the matrix  $H$  has not full rank. On the other hand, the iterative methods dedicated to solving variational inequalities are quite robust but with an extremely slow rate of convergence. To sum up, as far as we know there is no option that combines time efficiency and robustness. To try to answer to this question, we develop an open collection of discrete frictional contact problems called FCLIB (<http://fclib.gforge.inria.fr>) in order to offer a large library of problems to compare algorithms on a fair basis. In this work, this collection is solved with the software Siconos and its component Siconos/Numerics (<http://siconos.gforge.inria.fr>).

## Synchronization of mechanical systems with a geometric unilateral constraint (13:50)

Nathan van de Wouw (Eindhoven University of Technology, The Netherlands), M. Baumann, J.J.B. Biemond, R.I. Leine

This presentation addresses the synchronization problem of mechanical systems with a geometric unilateral constraint. The impacts of the individual systems, induced by the unilateral constraint, generally do not coincide even if the solutions are arbitrarily 'close' to each other. The mismatch in the impact time instants requires a careful choice of the distance function to allow an intuitively correct comparison of the discontinuous solutions. We propose a distance function induced by the quotient metric, which is based on an equivalence relation using the impact map. The distance function obtained in this way is continuous in time when evaluated along jumping solutions. Furthermore, the complexity of the distance function can be reduced significantly due to the property of monotonicity of the most commonly used impact laws. Based on the generic distance function, a Lyapunov function is constructed to investigate the synchronization problem for two identical one-dimensional mechanical systems. Sufficient conditions for the individual systems and their interaction are provided under which synchronization can be ensured. Furthermore, we present a (coupling) control law which ensures global synchronization, also in the presence of grazing trajectories and accumulation points (Zeno behavior). The results are illustrated using several numerical examples.

## Simulation of grasping tasks using the nonsmooth generalized- $\alpha$ method (15:00)

Olivier Brüs (University of Liège, Belgium)

In robotics, a grasping task involves a mechanical interaction between a robotic arm, its end-effector and the object to be grasped. In this work, the robot and the end-effector are modelled as a multibody system composed of a set of bodies and kinematic joints, which interact with the object through unilateral contact conditions with friction. Therefore, the equations of motion include a set of nonlinear bilateral and unilateral constraints, which can become redundant depending on the contact status.

Previously, we have developed the nonsmooth generalized- $\alpha$  method for mechanical systems with frictionless contact conditions. The aim of the method is to combine the robustness of the nonsmooth contact dynamics method with a second-order generalized- $\alpha$  scheme in order to capture accurately dynamic and vibration phenomena. Also, all constraints can be enforced simultaneously at position and velocity levels provided the definition of a suitable constraint activation strategy in a Newton semi-smooth algorithm. This means that the non-penetrability condition is enforced exactly together with an impact law at velocity level.

In this talk, we discuss the applicability of this method to grasping problems with constraint redundancy and friction phenomena. The activation status is more complex in this case as (i) the Lagrange multipliers are not uniquely defined, (ii) the transitions between non-contact, sliding contact and sticking contact states have to be handled properly.